Math 72 6.4 Rational Equations

Objectives

1) Solve rational equations by multiplying both sides of the = sign by the LCD to clear fractions

2) Solve applications that result in rational equations (see list next page)

3) Recognize and reject extraneous solutions

* can skip "solve by substitution" ↳ covered 8.6 only.
or do now for exposure only

Applications Resulting in Rational Equations

Objectives

1) Direct translation for numbers
   "reciprocal of x" $= \frac{1}{x}$

   "quotient of x and 5" $= \frac{x}{5}$

   "sum" or "difference" means use parentheses or entire quantity above or below a fraction bar

2) Proportions
   one ratio = another ratio
   align like quantities down and across, not diagonally to set up the proportion
   solve by cross-multiply

3) Work rates
   - Find the fractions done by each worker in one unit of time
     Total job = 5 hrs $\Rightarrow \frac{1}{5}$ of job in one hour
   - Add fractions for each worker
   - Find fraction done by all workers in one unit of time
     Total job = x min $\Rightarrow \frac{1}{x}$ of job in one minute
   - set sum of individual fractions = fraction together

4) $D=RT$
   - use a chart
   - If you have D and R, use $\frac{D}{R}=T$
   - If "same time" given, set $\frac{D_1}{R_1}=\frac{D_2}{R_2}$
   - If you have D and T, use $\frac{D}{T}=R$
   - If "same rate" given, set $\frac{D_1}{T_1}=\frac{D_2}{T_2}$

5) Random equation given in the problem.
   - check what units go with each variable
   - Use units on given numbers to identify which variable

Review
1) Solve rational equations by clearing fractions by multiplying all terms both sides by the lowest common denominator.
2) Check graph using x-intercept method
3) Solve by substitution.
4) Reject extraneous solutions by checking domains.

② Solve $\frac{4x}{5} + \frac{3}{2} = \frac{3x}{10}$ algebraically.

step 1: Find LCD = 10.

step 2: Multiply all terms both sides by LCD.
(We can multiply by something not = 1 because it's an equation!)

$$10 \cdot \frac{4x}{5} + 10 \cdot \frac{3}{2} = 10 \cdot \frac{3x}{10}$$

step 3: Cancel LCD with denominators before multiplying numerators

$$\overset{2}{\cancel{10}} \cdot \frac{4x}{\cancel{5}} + \overset{5}{\cancel{10}} \cdot \frac{3}{\cancel{2}} = \cancel{10} \cdot \frac{3x}{\cancel{10}}$$

$$2 \cdot 4x + 15 = 3x$$

$$8x + 15 = 3x$$

step 4: Identify the type of equation.
degree 1 = linear ⇒ solve by isolating the variable.

degree 2 = quadratic ⇒ Solve by factoring, quadratic formula, complete the square, or graphing.

Note: In Math 70 so far, we have only done factoring, so every quadratic equation in 6.5 can be solved by factoring.

step 5: Solve.

$$15 = -5x$$

$$\boxed{-3 = x}$$

EXPLORE

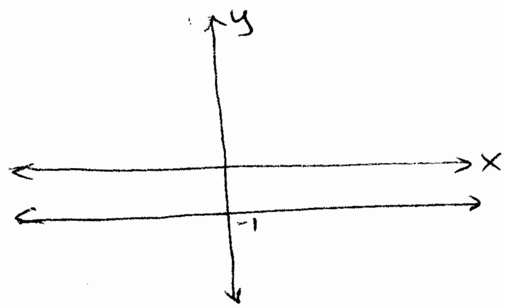
(3) Estimate solution of $\frac{x+6}{x-2} = \frac{2(x+2)}{x-2}$ by using the x-intercept graphing method.

step 1: Set equation = 0.

$$\frac{x+6}{x-2} - \frac{2(x+2)}{x-2} = 0$$

step 2: Graph in GC being very careful to use added parentheses.

$Y_1 = (x+6)/(x-2) - 2(x+2)/(x-2)$



step 3: Find x-coordinates where graph intersects the x-axis.

None – $\boxed{\text{No solution}}$

DO IT THIS WAY!

(3) Solve $\frac{x+6}{x-2} = \frac{2(x+2)}{x-2}$ algebraically.

step 1: Find the domain of each expression by solving equatio made by setting denominator = 0

$x-2=0$

$x \neq 2.$

step 2: Find LCD and multiply all terms on both sides.

$$\frac{(x-\cancel{2})(x+6)}{(x-\cancel{2})} = \frac{2(x+2)(x-\cancel{2})}{(x-\cancel{2})}$$

$x+6 = 2x+4$

$2 = x$

step 3: Reject extraneous solution that's not in domain.

$\cancel{x=2}$ $\boxed{\text{No SOLUTION}}$

4. Solve $\dfrac{2x}{2x-1} + \dfrac{1}{x} = \dfrac{1}{2x-1}$ $\quad x \neq 0, \; x \neq \frac{1}{2}$

LCD = $x(2x-1)$

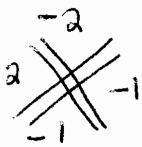
Multiply by LCD

$$\frac{2x}{\cancel{(2x-1)}} \cdot x\cancel{(2x-1)} + \frac{1}{\cancel{x}} \cdot \cancel{x}(2x-1) = \frac{1}{\cancel{(2x-1)}} \cdot x\cancel{(2x-1)}$$

$$2x \cdot x + 1 \cdot (2x-1) = x$$

$$2x^2 + 2x - 1 = x \qquad \text{simplify}$$

$$2x^2 + x - 1 = 0 \qquad \text{set} = 0$$



$$\underbrace{2x^2 + 2x}\underbrace{-x-1} = 0$$

$$2x(x+1) - 1(x+1) = 0$$

$$(x+1)(2x-1) = 0$$

$$\downarrow \qquad \downarrow$$

$\boxed{x = -1}$ $\quad \cancel{x = \frac{1}{2}}$ reject extraneous

An <u>extraneous solution</u> is a value found by correct work but which is not in the domain of one or more expressions.

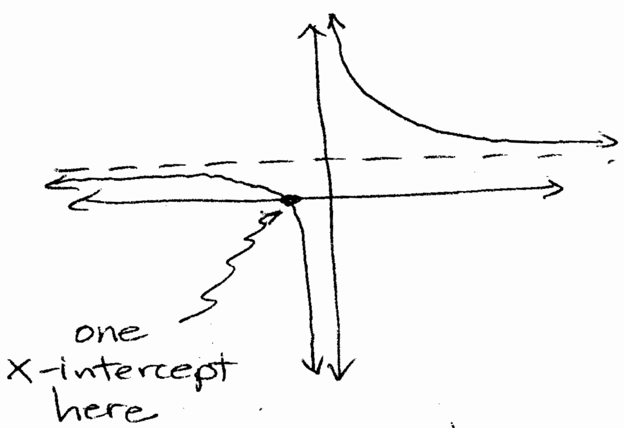
If we substitute this value, we get "undefined" instead of a true statement.

Correct work must reject all extraneous solutions by crossing them out and excluding them from the final answer, including writing "no solution" if all values were rejected.

(4) Solve by graphing

$$\frac{2x}{2x-1} + \frac{1}{x} = \frac{1}{2x-1}$$

$Y_1 = 2x / (2x-1) + 1/x - 1/(2x-1)$



To find by using graphing calculator (TI)

$\boxed{\text{2nd}}\boxed{\text{TRACE}}$ = CALC

2. Zero

Left bound? Move cursor to the left of x-int. $\boxed{\text{ENTER}}$

Right bound? Move cursor to the right of x-int $\boxed{\text{ENTER}}$

Guess? $\boxed{\text{ENTER}}$

$\boxed{x = -1}$ write x-coordinate only.

Solve by substitution

(5) $x^{-2} - 19x^{-1} + 48 = 0$

Notice $(x^{-1})^2 = x^{-2}$ makes this equation quadratic in form even though it's not a true quadratic.

Substitute $u = x^{-1}$ and factor/solve.

$$u^2 - 19u + 48 = 0$$

48

-3 -16

-19

1, 48
2, 24
3, 16

$$(u-3)(u-16) = 0$$

$$u = 3 \qquad u = 16$$

Replace u by x^{-1} again, solve again.

$$x^{-1} = 3 \qquad x^{-1} = 16$$

$$\frac{1}{x} = 3 \qquad \frac{1}{x} = 16$$

$$1 = 3x \qquad 1 = 16x$$

$$\frac{1}{3} = x \qquad \frac{1}{16} = x$$

$$\boxed{x = \frac{1}{3}, \frac{1}{16}}$$

(6) $\left(\frac{3}{x-1}\right)^2 + 2\left(\frac{3}{x-1}\right) + 1 = 0$ domain: $x \neq 1$

substitute $u = \frac{3}{x-1}$

$$u^2 + 2u + 1 = 0$$

$$(u+1)(u+1) = 0$$

$$u = -1$$

$$\frac{3}{x-1} = -1$$

$$3 = -(x-1)$$

$$3 = -x + 1$$

$$2 = -x$$

$$\boxed{-2 = x}$$

Solving Rational Equations

(6) #6 can be done by two methods:

$$\left(\frac{3}{x-1}\right)^2 + 2\left(\frac{3}{x-1}\right) + 1 = 0$$

Method 1: By substitution

$$u = \frac{3}{x-1} \Rightarrow u^2 = \left(\frac{3}{x-1}\right)^2$$

$$u^2 + 2u + 1 = 0$$

Factor

$$(u+1)(u+1) = 0$$

$$u = -1$$

Substitute back for u:

$$\frac{3}{x-1} = -1$$

Solve for x. Mult by x-1.

$$(x-1)\cdot\frac{3}{\cancel{(x-1)}} = -1(x-1)$$

$$3 = -x + 1$$

$$2 = -x$$

$$\boxed{x = -2}$$

Check $x \neq 1$ OK

Method 2. By multiplying.

$$\frac{9}{(x-1)^2} + \frac{6}{(x-1)} + 1 = 0$$

$$LCD = (x-1)^2$$

Multiply by LCD:

$$\frac{9\cancel{(x-1)^2}}{\cancel{(x-1)^2}} + \frac{6}{\cancel{(x-1)}}\cdot(x-1)^{\cancel{2}} + 1(x-1)^2 = 0(x-1)^2$$

$$9 + 6(x-1) + (x-1)^2 = 0$$

dist and FOIL

$$9 + 6x - 6 + x^2 - 2x + 1 = 0$$

combine

$$x^2 + 4x + 4 = 0$$

factor

$$(x+2)^2 = 0$$

Solve

$$\boxed{x = -2}$$

check

$x \neq 1$ OK

(7) Solve $\frac{x+z}{y} = \frac{6x+9y}{z}$ for x

cross-multiply to clear fractions:

$z(x+z) = y(6x+9y)$ dist

$xz + z^2 = 6xy + 9y^2$ find the x values

collect them on same side and other terms to other side

$xz - 6xy = 9y^2 - z^2$

$x(z-6y) = 9y^2 - z^2$ factor out GCF x

$$x = \frac{9y^2 - z^2}{z - 6y}$$

Rational expressions should be left factored

$$\boxed{x = \frac{(3y-z)(3y+z)}{(z-6y)}}$$

(9) $(4-x)^2 - 5(4-x) + 6 = 0$

$$u = 4 - x$$

$$u^2 - 5u + 6 = 0$$

$$(u-2)(u-3) = 0$$

$$u = 2 \qquad u = 3$$

$$4 - x = 2 \qquad 4 - x = 3$$

$$-x = -2 \qquad -x = -1$$

$$\boxed{x = 2 \qquad x = 1}$$

(10) Solve $\dfrac{z}{2z^2+3z-2} - \dfrac{1}{2z} = \dfrac{3}{z^2+2z}$

Note: Lots of twos and z's. This is a deliberate trap for bad handwriting!
Loop the 2's
Cross the z's.
Sharpen z's.
Round the 2's.

$$2z^2+3z-2=0 \qquad z \neq 0 \qquad z^2+2z=0$$

$$(2z-1)(z+2)=0 \qquad z(z+2)=0$$

$$z \neq \tfrac{1}{2}, -2 \qquad z \neq 0, 2$$

$$\text{LCD} = 2z(2z-1)(z+2)$$

$$\frac{z}{\cancel{(2z-1)}\cancel{(z+2)}} \cdot 2z\cancel{(2z-1)}\cancel{(z+2)} - \frac{1}{\cancel{2z}} \cdot \cancel{2z}(2z-1)(z+2) = \frac{3}{\cancel{z}\cancel{(z+2)}} \cdot 2\cancel{z}(2z-1)\cancel{(z+2)}$$

$$2z^2 - (2z-1)(z+2) = 6(2z-1)$$

$$2z^2 - (2z^2+3z-2) = 12z - 6$$

$$2z^2 - 2z^2 - 3z + 2 = 12z - 6$$

$$-3z + 2 = 12z - 6$$

$$8 = 15z$$

$$\boxed{\frac{8}{15} = z}$$

Hint: If you know how to find the LCD and multiply it, but you're not getting the right answer, write out your work with more detail and more neatly.

(11) $\dfrac{2x+3}{3x-2} = \dfrac{4x+1}{6x+1}$

$x \neq \frac{2}{3}, -\frac{1}{6}$

$LCD = (3x-2)(6x+1)$

$$\cancel{(3x-2)}(6x+1) \cdot \frac{(2x+3)}{\cancel{(3x-2)}} = \frac{(4x+1)}{\cancel{(6x+1)}} \cdot (3x-2)\cancel{(6x+1)}$$

$$(6x+1)(2x+3) = (4x+1)(3x-2)$$

FOIL!

$$12x^2+18x+2x+3 = 12x^2-8x+3x-2$$

$$20x+3 = -5x-2$$

$$25x = -5$$

$$\boxed{x = -\frac{1}{5}}$$

in domain ✓

(12) $\dfrac{2}{x-5} + \dfrac{1}{2x} = \dfrac{5}{3x^2-15x}$

$x \neq 5, 0$ $\qquad 3x(x-5)$

$LCD = 3 \cdot 2 \cdot x \cdot (x-5) = 6x(x-5)$

$$\frac{2}{\cancel{x-5}} \cdot 6x\cancel{(x-5)} + \frac{1}{\cancel{2x}} \cdot \overset{3}{\cancel{6x}}(x-5) = \frac{5}{\cancel{3x(x-5)}} \cdot \overset{2}{\cancel{6x(x-5)}}$$

$$12x + 3(x-5) = 10$$

$$12x + 3x - 15 = 10$$

$$15x = 25$$

$$x = \frac{25}{15}$$

$$\boxed{x = \frac{5}{3}}$$

in domain ✓

Extras Solve.

(13) $\frac{2x}{x-3} + \frac{6-2x}{x^2-9} = \frac{x}{x+3}$

$x \neq 3$ $(x+3)(x-3)$ $x \neq -3$

$x \neq 3, -3$

LCD $(x+3)(x-3)$

$\frac{2x}{\cancel{x-3}} \cdot (x+3)\cancel{(x-3)} + \frac{6-2x}{\cancel{(x-3)(x+3)}} \cdot \cancel{(x+3)(x-3)} = \frac{x}{\cancel{x+3}} \cdot \cancel{(x+3)}(x-3)$

$2x(x+3) + 6 - 2x = x(x-3)$

$2x^2 + 6x + 6 - 2x = x^2 - 3x$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$\boxed{x = -6, -1}$

all in domain ✓

quadratic (degree 2)
set = 0
factor
set factors = 0.

(14) $\frac{x^2-20}{x^2-7x+12} = \frac{3}{x-3} + \frac{5}{x-4}$

$(x-3)(x-4)$

$x \neq 3, 4$

LCD $= (x-3)(x-4)$

$\frac{x^2-20}{\cancel{(x-3)(x-4)}} \cancel{(x-3)(x-4)} = \frac{3}{\cancel{(x-3)}} \cancel{(x-3)}(x-4) + \frac{5}{\cancel{(x-4)}} (x-3)\cancel{(x-4)}$

$x^2 - 20 = 3(x-4) + 5(x-3)$

$x^2 - 20 = 3x - 12 + 5x - 15$

$x^2 - 20 = 8x - 27$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$\boxed{x = 7, 1}$

all in domain ✓

Examples

Solve.

1) Five divided by the sum of a number and 4, minus the quotient of 3 and the difference of the number and 4 is equal to 6 times the reciprocal of the difference of the number squared and 16. What is the number?

2) To estimate the number of people in Springfield, population 10,000, who have a swimming pool in their backyard, 250 people were interviewed. Of those polled, 108 had a swimming pool. How many people in the city might one expect to have a swimming pool? (Round to the nearest whole number, if necessary.)

3) A painter can finish painting house in 5 hours. Her assistant takes 7 hours to finish the same job. How long would it take for them to complete the job if they were working together?

4) A baker can decorate the day's cookie supply four times as fast as his new assistant. If together they can decorate the day's cookie supply in 16 minutes, how long does it take each one to do the job alone?

5) A car travels 400 miles on level terrain in the same amount of time it travels 160 miles on mountainous terrain. If the rate of the car is 30 miles per hour less in the mountains than on level ground, find its rate in the mountains.

6) In electronics, the relationship among the resistances R_1 and R_2 of two resistors wired in a parallel circuit and their combined resistance R is described by the formula $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$. If the combined resistance of two resistors wired in a parallel circuit is 2 ohms and one of the two resistances is 8 ohms, find the other resistance.

Extra problems

7) One pump can drain a pool in 9 minutes. When a second pump is also used, the pool only takes 4 minutes to drain. How long would it take the second pump to drain the pool if it were the only pump in use?

8) A cyclist bikes at a constant speed for 16 miles. He then returns home at the same speed but takes a different route. His return trip takes one hour longer and is 21 miles. Find his speed.

9) One conveyor belt can move 1000 boxes in 7 minutes. Another can move 1000 boxes in 10 minutes. If another conveyor belt is added and all three are used, the boxes are moved in 3 minutes. How long would it take the third conveyor belt along to do the same job?

10) A boat moves 9 km upstream in the same amount of time it moves 19 km downstream. If the rate of the current is 8 km per hour, find the rate of the boat in still water.

11) A recent advertisement claimed that 2 out of every 3 doctors recommend a certain herbal supplement to increase energy levels. If a local hospital employs 250 doctors, how many doctors would you expect to recommend the supplement? (Round to the nearest whole number, if necessary.)

12) Jim can run 5 miles per hour on level ground on a still day. One windy day, he runs 15 miles with the wind, and in the same amount of time runs 4 mils against the wind. What is the rate of the wind?

13) One way to gauge whether a person may need to gain or lose weight is to use the body-mass index, which is calculated using the formula $B = \frac{705w}{h^2}$, where w is the weight in pounds and h is the height in inches. Doctors recommend that body-mass index values fall between 19 and 25. If a person is 6 ft 6 in. tall and has a body-mass index of 24, what should his or her weight be? (Round to the nearest pound.)

14) Five divided by the difference of a number and 8 equals the quotient of 10 and the sum of the number and 4. Find the number.

15) Two times the reciprocal of a number equals 28 ties the reciprocal of 35. Find the number.

16) Mark and Rachel both work for Smith Landscaping Company. Mark can finish a planting job in 2 hours, while if takes Rachel hours 4 hours to finish the same job. If Mark and Rachel will work together on the job, and the cost of labor is $40 per hour, what should the labor estimate be? (Round to the nearest cent, if necessary.)

17) In a race, Car A starts 1 mile behind Car B. Car A is traveling at 45 miles per hour, while Car B is traveling at 40 miles per hour. How long will it take for Car A to overtake Car B?

① Set up and solve – direct translate

$$\frac{5}{x+4} - \frac{3}{x-4} = 6 \cdot \frac{1}{x^2-16}$$

$$\frac{5}{x+4} - \frac{3}{x-4} = \frac{6}{(x+4)(x-4)}$$

domains: $x \neq -4$ $\quad x \neq 4 \quad$ $x \neq -4, 4$

$LCD = (x+4)(x-4)$

$$\frac{5}{\cancel{(x+4)}} \cdot \cancel{(x+4)}(x-4) - \frac{3}{\cancel{(x-4)}}(x+4)\cancel{(x-4)} = \frac{6}{\cancel{(x+4)}\cancel{(x-4)}} \cdot \cancel{(x+4)}\cancel{(x-4)}$$

$$5(x-4) - 3(x+4) = 6$$

$$5x - 20 - 3x - 12 = 6$$

$$2x - 32 = 6$$

$$2x = 38$$

$$\boxed{x = 19}$$

② Proportion

$$\begin{array}{rcl} \text{pool/survey} \rightarrow & \dfrac{108}{250} = \dfrac{x}{10{,}000} & \leftarrow \text{pool/population} \\ \text{total/survey} \rightarrow & & \leftarrow \text{total/population} \end{array}$$

Check across and up & down that similar quantities are aligned.

Solve by cross-multiply:

$$(108)(10{,}000) = 250x$$

$$1080000 = 250x$$

$$\frac{1080000}{250} = x$$

$x = \boxed{4320 \text{ have pools}}$

(3) Setup and solve — work rate

painter: 5 hrs total job alone $\Rightarrow$ $\frac{1}{5}$ of job in one hour

assistant: 7 hrs total job alone $\Rightarrow$ $\frac{1}{7}$ of job in one hour

together: x hrs total $\Rightarrow$ $\frac{1}{x}$ of job in one hour

These fractions are called work rates.

Write equation using work rates, which all refer to one hour of work:

$$\underbrace{\frac{1}{5}}_{\substack{\text{of job}\\\text{done}\\\text{by}\\\text{painter}\\\text{alone}}} + \underbrace{\frac{1}{7}}_{\substack{\text{of job}\\\text{done}\\\text{by}\\\text{assistant}\\\text{alone}}} = \underbrace{\frac{1}{x}}_{\substack{\text{of job}\\\text{done}\\\text{together}}}$$

We expect x to be less than 5 because the painter alone takes 5 hrs and the assistant's help only shortens the time needed.

LCD $= 35x$

$$\frac{1}{\cancel{5}} \cdot \overset{7}{\cancel{35}}x + \frac{1}{\cancel{7}} \cdot \overset{5}{\cancel{35}}x = \frac{1}{\cancel{x}} \cdot 35\cancel{x}$$

$$7x + 5x = 35$$

$$12x = 35$$

$$x = \frac{35}{12}\text{ hrs} = \boxed{2\tfrac{11}{12}\text{ hrs}} = \boxed{2.91\overline{6}\text{ hrs}}$$

best answer

acceptable answer with all decimals and repeat bar

$\boxed{\frac{35}{12}\text{ hrs}}$ ← ok, but harder to compare to 5 and 7 to check if answer makes sense.

(3) Method 2: Set up and solve — work rate using units analysis to compare this question to a D=RT problem.

painter: 5 hrs per 1 job = rate $\frac{5 \text{ hrs}}{1 \text{ job}}$ or $\frac{1 \text{ job}}{5 \text{ hrs}}$

amount of work done by painter = (Rate)·(Time)

$$= \frac{1 \text{ job}}{5 \cancel{\text{hrs}}} \cdot t \cancel{\text{hrs}} = \frac{t}{5} \text{ job}$$

assistant: 7 hrs per 1 job = rate $\frac{5 \text{ hrs}}{1 \text{ job}}$ or $\frac{1 \text{ job}}{5 \text{ hrs}}$

amount of work done by assistant = (Rate)·(Time)

$$= \frac{1 \text{ job}}{7 \cancel{\text{hrs}}} \cdot t \cancel{\text{hrs}} = \frac{t}{7} \text{ job}$$

together: t hrs per 1 job = rate $\frac{t \text{ hrs}}{1 \text{ job}}$ or $\frac{1 \text{ job}}{t \text{ hrs}}$.

amount of work done together = (Rate)·(Time)

$$= \frac{1 \text{ job}}{t \cancel{\text{hrs}}} \cdot t \cancel{\text{hrs}} = \frac{t}{t} \text{ job}$$

$$= 1 \text{ job}$$

Equation: (work done by painter) + (work done by assistant) = (work done together)

$$\frac{t}{5} \text{ job} + \frac{t}{7} \text{ job} = 1 \text{ job}$$

or $\frac{t}{5} + \frac{t}{7} = 1$

LCD = 5·7 = 35

$$35 \cdot \frac{t}{5} + 35 \cdot \frac{t}{7} = 35 \cdot 1$$

$$7t + 5t = 35$$

$$12t = 35$$

$t = \boxed{\frac{35}{12} \text{ hrs}}$ same as Method 1.

(4) work rate

baker 4 times as fast as assistant.
Whose work time (total) is shorter?
$\Rightarrow$ The baker's.

Let baker's work time $= x$ — fraction done in min $\frac{1}{x}$

assistant's work time $= 4x$ — " " " " $\frac{1}{4x}$

$$\underset{\text{baker}}{\frac{1}{x}} + \underset{\text{asst}}{\frac{1}{4x}} = \frac{1}{16}$$

fractions done in one minute

solve for x, then find $4x$ also.

LCD $= 16x$

$16 + 4 = x$

$x = 20$ min

$4x = 80$ min

less time $\Rightarrow$ faster work

$$\boxed{\begin{array}{l} x = \text{baker} = 20 \text{ mins} \\ 4x = \text{asst} = 80 \text{ min} \end{array}}$$

Caution: "faster" means "less time"
So faster person has a lower number.

$D = R \cdot T$ relates distance to rate (velocity or speed) and time.

$\frac{D}{R} = \frac{R \cdot T}{R}$ dividing both sides by R gives

$\frac{D}{R} = T.$ Use this if "same time".

$\frac{D}{T} = \frac{R \cdot T}{T}$ dividing both sides by T gives

$\frac{D}{T} = R.$ Use this if "same rate".

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(5) $D = R \cdot T \Rightarrow T = \frac{D}{R}$

	D	R	T
level	400	R+30	$\frac{400}{R}$
mtn	160	R	$\frac{160}{R-30}$

can also use $\frac{R}{R-30}$

but R isn't the answer to the question!

$$\frac{400}{R+30} = \frac{160}{R}$$

same time $\Rightarrow$ use $\frac{D}{R} = T$

(5) cont LCD = R(R+30) or cross multiply

$$400R = 160(R+30)$$
$$400R = 160R + 4800$$
$$240R = 4800$$
$$\boxed{R = 20 \text{ mph in mtns}}$$

Method 2: 2 variables

	D	= R	· T
level	400	R+30	T
mtns	160	R	T

$$\begin{cases} 400 = T(R+30) \\ 160 = R \cdot T \end{cases}$$

$$T = \frac{160}{R}$$

$$400 = \frac{160}{R}(R+30)$$

$$400 = 160 + \frac{4800}{R}$$

$$240 = \frac{4800}{R}$$

$$240R = 4800$$

$$\boxed{R = 20 \text{ mph}}$$

Note: units analysis

$$(R+30)\frac{mi}{\cancel{hr}} \cdot (T)\cancel{hr} = T(R+30) \text{ miles} = 400 \text{ miles}$$

$$(R)\frac{mi}{\cancel{hr}} \cdot (T)\cancel{hr} = R \cdot T \text{ miles} = 160 \text{ miles}$$

(6) Use given formula
R, R_1 and R_2 are three different variables

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

⇐ 3 variables in formula means 2 numbers will be given so we can solve for one.

R = combined resistance given 2

separate resistances

R_1 given (or R_2 if you prefer)

$$\frac{1}{2} = \frac{1}{8} + \frac{1}{R_1}$$

LCD of 2, 8, and R_1 is $8R_1$.

$$\frac{1}{2} \cdot 8R_1 = \frac{1}{8} \cdot 8R_1 + \frac{1}{R_1} \cdot 8R_1$$ mult all by LCD

$$4R_1 = R_1 + 8$$ divide out/ cancel common factors

$$3R_1 = 8$$ subtract R_1 both sides

$$\boxed{R_1 = \frac{8}{3} \text{ ohms}}$$

(7)

$$\underbrace{\frac{1}{9}}_{\text{1st pump fraction}} + \underbrace{\frac{1}{x}}_{\text{2nd pump fraction}} = \underbrace{\frac{1}{4}}_{\text{both pumps fraction}}$$

LCD of 9, x, and 4 is 36x

$$\frac{1}{9} \cdot 36x + \frac{1}{x} \cdot 36x = \frac{1}{4} \cdot 36x$$ mult all by LCD

$$4x + 36 = 9x$$ subtract 4x

$$36 = 5x$$

$$\frac{36}{5} = x$$

$$x = \boxed{\frac{36}{5} \text{ min}} = \boxed{7.2 \text{ min}}$$

⑧ $D = R \cdot T$

D	R	T
16	$\frac{16}{T}$	T
21	$\frac{21}{T+1}$	$T+1$

⇑

Same rate $\Rightarrow R = \frac{D}{T}$

⇓

Set rates equal $\qquad \frac{16}{T} = \frac{21}{T+1}$

$16(T+1) = 21(T)$ cross-multiply

$16T + 16 = 21T$ dist

$16 = 5T$ subtract $16T$

time is $3\frac{1}{5}\text{hr} = \frac{16}{5} = T$

But question asked for rate!

$$R = \frac{16}{T} = \frac{16}{(16/5)} = 16 \div \frac{16}{5} = 16 \cdot \frac{5}{16} = \boxed{5\ \frac{\text{mi}}{\text{hr}}}$$

Option 2:

$D = R \cdot T$

D	R	T	
16	R	$\frac{16}{R}$	← T shorter time
21	R	$\frac{21}{R}$	← T+1 longer time

$$\underbrace{\frac{21}{R}} = \underbrace{\frac{16}{R}} + 1 \qquad \text{LCD} = R$$

to get longer time, add 1 to shorter time

$21 = 16 + R$

$5 = R$

$\boxed{R = 5 \text{ mph}}$

Option 3:

$D = R \cdot T$

D	R	T
16	R	T
21	R	T+1

$\left.\begin{array}{l} 16 = R \cdot T \\ 21 = R(T+1) \end{array}\right\}$ solve system by substitution

$R = \frac{16}{T}$ subst into $21 = \frac{16}{T}(T+1)$ mult by T

$21T = 16(T+1)$ continue as before.

⑨ "1000 boxes" = 1 job. The number 1000 is not needed.

$$\frac{1}{7} + \frac{1}{10} + \frac{1}{x} = \frac{1}{3}$$

$\frac{1}{7}$	$\frac{1}{10}$	$\frac{1}{x}$	$\frac{1}{3}$
fraction done by 1st	2nd	3rd	all 3 together

LCD of $7, 10, x, 3 = 210x$

$$\frac{1}{7}\cdot 210x + \frac{1}{10}\cdot 210x + \frac{1}{x}\cdot 210x = \frac{1}{3}\cdot 210x$$

$$30x + 21x + 210 = 70x$$

$$51x + 210 = 70x$$

$$210 = 19x$$

$$\frac{210}{19} = x$$

$$x = \boxed{11\tfrac{1}{19}\text{ min}}$$

⑩ $D = R\cdot T$

Going upstream is against the current
The current slows us down — RATE (STILL WATER) − CURRENT RATE

Going downstream is with the current
The current speeds us up. — RATE (STILL WATER) + CURRENT RATE

	D	= R	· T
upstream	9	$R-8$	$\frac{9}{R-8}$
downstream	19	$R+8$	$\frac{19}{R+8}$

⇑ same time

use $\frac{D}{R} = T$

equation: $\frac{9}{R-8} = \frac{19}{R+8}$

(10) con't

LCD = $(R-8)(R+8)$ or cross-multiply

$9(R+8) = 19(R-8)$

$9R + 72 = 19R - 152$

$224 = 10R$

$\boxed{22.4 \text{ kph} = R}$

(11) $\frac{2}{3}$ of doctors

$\frac{2}{3}$ of 250

$\frac{2}{3} \cdot 250 = \frac{500}{3} = 166.\overline{6}$

round to the nearest whole doctor

$\boxed{167 \text{ doctors}}$

(12) $D = R \cdot T$

Going against wind
The wind slows us down

RATE (no wind) − WIND RATE

Going with wind
The wind speeds us up

RATE (no wind) + WIND RATE

	D	= R	· T
against	4	$5-R$	$\frac{4}{5-R}$
with	15	$5+R$	$\frac{15}{5+R}$

⇑ same time

use $\frac{D}{R} = T$

equation: $\frac{4}{5-R} = \frac{15}{5+R}$

LCD or cross-multiply

$4(5+R) = 15(5-R)$

$20 + 4R = 75 - 15R$

$19R = 55$

$R = \frac{55}{19} = \boxed{2\frac{17}{19} \text{ mph}}$

(13) Use given formula

$$B = \frac{705w}{h^2}$$

h = height in inches
height given 6 ft 6 in. = $6 \cdot 12 + 6 = 78$ inches
B = body mass index = 25
find weight. ⇒ solve for w.

$$24 = \frac{705w}{(78)^2}$$

$$24 = \frac{705w}{6084}$$ simplify 78^2

$$24(6084) = 705w$$ mult by 6084

$$146016 = 705w$$

$$\frac{146016}{705} = w$$ isolate w.

$$w = \frac{48672}{235}$$ >frac.

$$w \approx 207.1148936$$

$$\boxed{w \approx 207 \text{ lbs}}$$ round to nearest pound

(14) Direct translation

$$\frac{5}{x-8} = \frac{10}{x+4}$$ LCD $(x-8)(x+4)$ or cross-multiply

$$5(x+4) = 10(x-8)$$

$$5x + 20 = 10x - 80$$ dist

$$100 = 5x$$

$$\boxed{20 = x}$$

(15) Direct translation

$$2 \cdot \frac{1}{x} = 28 \cdot \frac{1}{35}$$

$$\frac{2}{x} = \frac{28}{35}$$ cross multiply

$$70 = 28x$$ isolate

$$x = \frac{70}{28} = \boxed{\frac{5}{2}}$$ reduce

(16)

$$\frac{1}{2} + \frac{1}{4} = \frac{1}{x}$$

Mark Rachel Together

← fractions done in one hour.

$LCD = 4x$

$$\frac{1}{2}\cdot 4x + \frac{1}{4}\cdot 4x = \frac{1}{x}\cdot 4x$$

$$2x + x = 4$$

$$3x = 4$$

$$x = \frac{4}{3}\text{ hr} = 1\tfrac{1}{3}\text{ hr}$$

Labor = \$40 per hour per person ⇒ \$80 per hour for both people

Cost of labor for $\frac{4}{3}$ hr $= \frac{4}{3} \times 80 = \frac{320}{3} \approx \boxed{\$106.67}$

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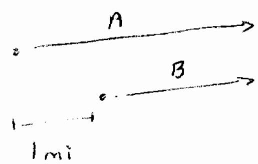
(17)

	D	= R	· T
A	45T	45	T
B	40T	40	T



same direction ⇒ subtract distances
(not a rational equation)

$$45T - 40T = 1 \text{ mi}$$

$$5T = 1$$

$$\boxed{T = \frac{1}{5}\text{ hr}}$$

$$\frac{1}{5}\text{ hr} \cdot \frac{60\text{ min}}{1\text{ hr}} = \boxed{12 \text{ min}}$$

Extra A cyclist bikes at a constant speed for 23 miles. He returns home at the same speed but takes a different route. His return trip takes one hour longer and is 28 miles. Find his speed.

Alternate approach to the one in your class notes:
(see 6.8 handout)

D	= R	· T
23	$\frac{23}{T}$	T
28	$\frac{28}{T+1}$	$T+1$

Many of you wanted to use variables for time instead of for rate.

$$D = R \cdot T$$

means

$$\frac{D}{T} = R$$

"Same speed" means:

$$\frac{23}{T} = \frac{28}{T+1}$$

Cross multiply

$$23(T+1) = 28T$$

$$23T + 23 = 28T$$

$$23 = 5T$$

$$4.6 \text{ hrs} = 4\tfrac{3}{5} \text{ hrs} = \frac{23}{5} \text{ hrs} = T$$

⟸ This is the time to travel the 23 miles, not answer to question.

$$R = \frac{23}{T}$$

$$= \frac{23}{4.6}$$

⟸ To get speed, must plug back in.

$$\boxed{R = 5 \text{ mph}}$$